

Family Unification, Exotic States and Magnetic Monopoles

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Abstract

The embedding in $SU(4) \times SU(3) \times SU(3)$ of the well studied gauge groups $SU(4) \times SU(2) \times SU(2)$ and $SU(3) \times SU(3) \times SU(3)$ naturally leads to family unification as opposed to simple family replication. An inescapable consequence is the predicted existence of (exotic) color singlet states that carry fractional electric charge. The corresponding magnetic monopoles carry multiple Dirac magnetic charge, can be relatively light ($\sim 10^7 - 10^{13} GeV$), and may be present in the galaxy not far below the Parker bound.

Much work has been done in trying to extract standard model physics from string theory, beginning with Calabi-Yau compactifications of the heterotic string, which yield E_6 type GUT theories, where holomorphic deformations, Wilson loops, etc. can be used to reduce the gauge symmetry, and continuing today with orbifolding of type IIB strings on $AdS_5 \otimes S^5$ to produce conformal field theories (CFT s) with gauge groups $\prod_i SU(Nd_i)$ and bifundamental matter [1], [2]. Here we take a bottom-up approach and consider a model that is likely to be deriveable from orbifolded type IIB strings, but focus more on the physics that will result. The model we study contains aspects of both E_6 and CFT type string theory compactifications, and leads to a remarkably rich phenomenology. It is well known that the Pati-Salam (PS) model [3] and the Trinification [4] (TR) model are both contained in E_6 Grand Unification [5], [6]. We will provide another covering of PS and TR which does not embed in E_6 , but instead *requires* 3 families and is perhaps the minimal such example of a model with these properties. After a brief review of PS and TR, we present our model and then consider some of its consequences.

The PS model has gauge group $G_{PS} = SU(4) \times SU_L(2) \times SU_R(2)$, and fermion families in bifundamental representations

$$(4, 2, 1) + (\bar{4}, 1, 2) \tag{1}$$

This model embeds in $SO(10)$, where the fermions are then all contained in a **16**. Adding a **10** + **1** of fermions then allows unification into E_6 , where the fermions are then in a **27**. Various approaches to symmetry breaking have been studied, but we need not be concerned with these details for now. Note that this model is anomaly free for a single family, so a full three-family model is gotten simply by the inclusion of two more families. TR also has fermions in bifundamental representations of the gauge group $G_{TR} = SU(3) \times SU(3) \times SU(3)$:

$$(3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3}) \tag{2}$$

As this group is already a maximal subgroup of E_6 and (2) already contains 27 states, the unification into E_6 is gotten simply by adding the necessary gauge generators to extend $SU^3(3)$ to E_6 . Again, a single fermion family is anomaly free on its own, so we must add two families to agree with phenomenology.

The smallest group that contains both the SP model and TR is not E_6 but $G = SU(4) \times SU(3) \times SU(3)$, which has 31 generators and is rank 7. Insisting on fermions that fall into bifundamentals, the representations to consider are $(4, \bar{3}, 1)$, $(\bar{4}, 1, 3)$, and $(1, 3, \bar{3})$. We can not take one of each to form a family, since this would be anomalous. The minimal anomaly free choice is

$$3(4, \bar{3}, 1) + 3(\bar{4}, 1, 3) + 4(1, 3, \bar{3}) \quad (3)$$

If we break $SU(4)$ to $SU(3)$, then (3) becomes

$$3[(3, \bar{3}, 1) + 3(\bar{3}, 1, 3) + 3(1, 3, \bar{3})] + (1, \bar{3}, 1) + (1, 1, 3) + (1, 3, \bar{3}) \quad (4)$$

which contains three TR families plus a few additional particles. Hence the simplest anomaly free chiral bilinear representation [*i.e.*, (3)] contains three families (*i.e.*, this is a true family unification) instead of one as for either PS(1), TR(2), or the **27** of E_6 (where the second and third families are gotten from merely replicating the first). To further analyze this $SU(4) \times SU(3) \times SU(3)$ model [7] (334-model), we must consider the spontaneous symmetry breaking (SSB), and the charge assignments this leads to, plus the implication of the “extra” fermions. We will find there exist fractional charged color singlets [8], [9] in the model, and hence the minimal monopole charge in the model will be the inverse of the minimal fraction times the Dirac charge.

The possibilities for embedding color and weak isospin of the standard model gauge group in $SU(4) \times SU(3) \times SU(3)$ are:

- (i) Embed $SU_C(3)$ in one $SU(3)$ and $SU_W(2)$ in the other $SU(3)$.
- (ii) Embed $SU_C(3)$ in an $SU(3)$ and $SU_W(2)$ in $SU(4)$.
- (iii) Embed $SU_C(3)$ in the $SU(4)$ and $SU_W(2)$ in an $SU(3)$.

Other embeddings are equivalent except for embedding $SU_C(3)$ and/or $SU_W(2)$ in some diagonal subgroup within the 334-model. However, this leads to vectorlike fermions, and we need not pursue this possibility any further.

The embedding of weak hypercharge is more complicated. Consider the breaking $SU(4) \times SU_L(3) \times SU_R(3) \rightarrow SU(4) \times [SU_L(2) \times U_A(1)] \times [SU_R(2) \times U_B(1)]$. If we then break $U_A(1)$ and $U_B(1)$ completely, the hypercharge must be $Y = T_{3R} + (B - L)$, where T_{3R} is the diagonal generator of $SU_R(2)$, and $B - L$ generates the $U(1)$ that is in $SU(4)$ but not in $SU_C(3)$. However, there are other possibilities for the embedding of $U_Y(1)$. These are similar, and in some cases equivalent, to the well-known flipped models [10], [11]. One obvious choice is to break $SU(4)$ to $SU_C(3)$ and then Y could be the trinification choice from $SU_L(3) \times SU_R(3)$. Similarly, trinification has a standard hypercharge assignment, but this could be flipped to a Pati-Salam charge assignment. Also moving $SU_W(2)$ from $SU(4)$ to an $SU(3)$ of the 334-model corresponds to an isoflipped model [12]. (There are even more choices, but they will be described elsewhere.) Here we restrict ourselves to the standard hypercharge embeddings, but keep in mind that flipping may offer other opportunities.

Returning to the standard Pati-Salam version of the 334-model, we find on breaking G to G_{PS} the fermions become

$$3[(4, \bar{2}, 1) + (4, \bar{1}, 1)] + 3[(\bar{4}, 1, 2) + (\bar{4}, 1, 1)] + 4[(1, 2, \bar{2}) + (1, 2, 1) + (1, 1, \bar{2}) + (1, 1, 1)] \quad (5)$$

At this stage only the three families remain chiral, while the extra (exotic) vectorlike fermions obtain masses from Higgs VEVs at the G breaking scale. Now, breaking to the standard model $G_{SM} = SU_C(3) \times SU_W(2) \times U_Y(1)$, we find

$$\begin{aligned} & 3[(3, 2)_{\frac{1}{6}} + (1, 2)_{-\frac{1}{2}} + (3, 1)_{\frac{1}{6}} + (1, 1)_{-\frac{1}{2}}] \\ & + 3[(\bar{3}, 1)_{\frac{1}{3}} + (\bar{3}, 1)_{-\frac{2}{3}} + (1, 1)_1 + (1, 1)_0 + (\bar{3}, 1)_{-\frac{1}{6}} + (1, 1)_{\frac{1}{2}}] \\ & + 4[(1, 2)_{-\frac{1}{2}} + (1, 2)_{\frac{1}{2}} + (1, 2)_0 + (1, 1)_{-\frac{1}{2}} + (1, 1)_{\frac{1}{2}} + (1, 1)_0.] \end{aligned}$$

As expected, we are left with three standard families, plus three right-handed neutrinos, from the three Pati-Salam families. In addition we have the extra states:

$$Q_E = 3[(3, 1)_{\frac{1}{6}} + (\bar{3}, 1)_{-\frac{1}{6}}] + 4[(1, 2)_{\frac{1}{2}} + (1, 2)_{-\frac{1}{2}}] + 7[(1, 1)_{\frac{1}{2}} + (1, 1)_{-\frac{1}{2}}] \\ + 4(1, 2)_0 + 4(1, 1)_0.$$

Once color is confined, we see from Q_E that electric charge is quantized in units of $\frac{1}{2}$. So any magnetic monopoles that exist in the model must have minimum charge two from the Dirac quantization condition.

In the case of trinification, an interesting subtlety arises on breaking $SU(4)$ to $SU(3)$. With the standard trinification charge assignments, we will find massless charged quarks and leptons. To avoid the resulting conflict with phenomenology, we must add a few additional states at the 334 level. Let us see how this works. Two equivalent cases must be considered: (1) $SU_C(3)$ embedded in $SU(4)$, or (2) $SU_C(3)$ identified with an $SU(3)$ of the 334-model. In both cases at the trinification level we have fermions as in (4). For case 1, we have the three standard families plus

$$R_E = 3(1, \bar{3}, 1) + 3(1, 1, 3) + (1, 3, \bar{3}) \quad (6)$$

under $SU_C(3) \times SU_L(3) \times SU_R(3)$. Hence all the extra states are leptonic. Then for $SU_L(3) \times SU_R(3) \rightarrow SU_L(2) \times U_L(1) \times U_R(1)$ where we identify $U_R(1)$ with the diagonal generator $Y_R = \text{diag}(1, 1, -2)$ of $SU_R(3)$ and likewise $U_L(1)$ is generated by $Y_L = \text{diag}(1, 1, -2)$ of $SU_L(3)$, we can choose the hypercharge to be $Y = \frac{1}{6}Y_L + \frac{1}{3}Y_R$. The families just have the standard **27** of E_6 charges, while the new leptons are

$$5(1, 2)_{-\frac{1}{6}} + (1, 2)_{\frac{5}{6}} + 10(1, 1)_{\frac{1}{3}} + 5(1, 1)_{-\frac{2}{3}} \quad (7)$$

As these states are still chiral, the only way to give them mass would be with a VEV from an electrically charged Higgs. As this must obviously be avoided, the alternative is to arrange these particles to be vectorlike by adding the conjugate, but anomaly free chiral multiplets

$$\bar{R}_E = 3(1, 3, 1) + 3(1, 1, \bar{3}) + (1, \bar{3}, 3) \quad (8)$$

at the 334 level. In this case, upon breaking $G \rightarrow G_{TR}$ at the scale $\langle \phi \rangle$, the chiral families stay massless while the extra fermions acquire mass terms of the form $h \langle \phi \rangle \bar{R}_E R_E$ where

h is a typical Yukawa coupling constant. Hence, the fractionally charged leptons become heavy compared to the family fermions. Let us summarize the extra vectorlike leptons. There are five doublets with electric charges $\pm\frac{1}{3}$ and $\mp\frac{2}{3}$, two doublets with electric charge $\pm\frac{1}{3}$ and $\pm\frac{4}{3}$, ten singlets with $\pm\frac{1}{3}$ charges, and five singlets with $\mp\frac{2}{3}$ charges. The minimal monopole charge is three.

For case 2, some of the extra states will be colored. In terms of $SU_C(3) \times SU_L(3) \times SU_R(3)$, they are

$$S_E = 3(3, 1, 1) + 3(1, 1, \bar{3}) + (\bar{3}, 1, 3) \quad (9)$$

The hypercharge is unchanged (we continue to ignore flipping and other possible charge assignments) from case (1) (it is still $Y = \frac{1}{6}Y_L + \frac{1}{3}Y_R$), so we find:

$$S_E = 3(3, 1)_0 + 3(1, 2)_{-\frac{1}{3}} + 3(1, 1)_{\frac{2}{3}} + (\bar{3}, 2)_{\frac{1}{3}} + (\bar{3}, 1)_{-\frac{2}{3}} \quad (10)$$

Again we must add conjugate states

$$\bar{S}_E = 3(\bar{3}, 1, 1) + 3(1, 1, 3) + (3, 1, \bar{3}) \quad (11)$$

and this allows masses for the exotics at a higher scale than family masses.

The symmetry breaking scales of $SU(4) \times SU(2) \times SU(2)$ and $SU(3) \times SU(3) \times SU(3)$ determine the mass of the associated magnetic monopoles. In this context it is important to recall the absence of gauge boson mediated proton decay in these models. Proton decay can still proceed through higgs/higgsino exchange, as well as via higher dimension operators. It is relatively straight forward, however, to construct models based on G_{PS} and G_{TR} such that these processes are forbidden, as a consequence say of an ‘accidental’ baryon number symmetry. This opens up the possibility that G_{PS} and G_{TR} can be broken at scales considerably below the conventional grand unification scale $M_{GUT} \sim 10^{16} \text{ GeV}$. An example based on D-branes in Type I string theory was recently provided for the G_{PS} symmetry [13]. With the standard embedding of $SU(3)_C \times SU(2) \times U(1)$, the symmetry breaking scale of G_{PS} turns out to be $M_{PS} \sim 10^{12} - 10^{13} \text{ GeV}$, with the corresponding string scale $\gtrsim M_{PS}$.

Thus, monopoles with mass $\sim 10^{13} - 10^{14}$ GeV are expected in this class of models. An even more suggestive result is the Pati-Salam type model based on CFT obtained from orbifolded type IIB strings [14], [15]. Here the unification is in the $100TeV$ range, where this gives $\sin^2\theta = .23$ and other intriguing phenomenology [16]. There is no reason why analogous considerations cannot be carried out for the trinification scheme and, by extension, for the gauge symmetry of special interest here $SU(4) \times SU(3) \times SU(3)$. The multiply charged monopoles of the theory will have mass $M \sim 10^7 GeV$, *i.e.*, in the preferred range of interest if they are to be candidates for high energy cosmic ray primaries [17], [18]. We expect the 334-model to have a similar unification scale with resulting exotic (fractionally charged) leptons and/or hadrons, and we expect their masses to be near this unification scale, so they are of interest as dark matter candidates [19], [20].

Given that monopoles of mass $\sim 10^{13} - 10^{14}$ GeV (or perhaps even much lighter) can arise in realistic models, it is important to ask: Can primordial monopoles survive inflation? If the underlying theory is non-supersymmetric, an inflationary scenario which dilutes but does not completely wash away intermediate mass monopoles was developed in ref [21]. Note that the D-brane scenario discussed above gives rise to non-supersymmetric $SU(4)_C \times SU(2) \times SU(2)$, so the discussion in ref [21] may be relevant. The monopole flux can be close to the Parker bound of order $10^{-16} cm^{-2} s^{-1} sr^{-1}$. In the orbifolded scheme, the SSB scale where the monopoles get their masses can be below the inflation scale. Hence the monopoles can exist in interesting densities (near the Parker bound) depending on details of the SSB phase transitions. For the supersymmetric case, dilution of monopoles can be achieved by thermal inflation [22], [23] followed by entropy production. A scenario in which thermal inflation is associated with the breaking of the $U(1)$ axion symmetry was recently developed in ref [24].

In summary, the $SU(4) \times SU(3) \times SU(3)$ models we are advocating provides a natural family unification while avoiding proton decay and giving rise to both (exotic) fractionally charged color singlets and corresponding multiply charged magnetic monopoles [25] with masses that can be well below $M_{GUT} \sim 10^{16} GeV$, perhaps as light as $\sim 10^7 GeV$, (Note that

in $SU(5)$ the lightest monopole has mass of $\sim 10^{17} GeV$, and carries one unit of magnetic charge [26].) The exotic states are heavy (greater than a few TeV), but may be in the range explored by accelerators in the next decade.

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